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Fluctuation-induced magnetotransport of superconductors in the quasiballistic regime

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Abstract. We study the fluctuation-induced magnetotransport of a two-dimensional superconductor in the quasiballistic regime, where $\xi_{GL}(T) \ll \ell$ (ℓ is the electron mean free path and $\xi_{GL}(T)$ is the Ginzburg-Landau coherence length). The magnetoconductivity is evaluated in the nonlocal fluctuation regime thereby extending the existing theory valid in the local limit. We show that the Maki-Thompson (MT) and density-of-states (DOS) contributions strongly compensate each other and their sum is negligible in comparison with the Aslamazov-Larkin (AL) term. The hierarchy of the fluctuation contributions to the magnetoconductivity in the high-field limit is also qualitatively discussed.

PACS. 72.10.-d Theory of electronic transport; scattering mechanisms – 71.30.+h Metal-insulator transitions and other electronic transitions – 71.10.Ay Fermi-liquid theory and other phenomenological models

1 Introduction

Recently it has been demonstrated [1,2] that the quasiballistic character of the electron motion in a clean superconductor can significantly change the traditional picture of the fluctuation phenomena manifestation above the critical temperature. In particular, in the regime where the electron mean free path ℓ considerably exceeds the effective size of the fluctuating Cooper pair $\xi_{GL}(T) = \xi_0/\sqrt{\epsilon}$ ($\epsilon = (T - T_c)/T_c$, $\xi_0^2 = 7\zeta(3)\, v_F^2/32\,(\pi T)^2$, $\zeta(3) \approx 1.21$ is the Riemann's $\zeta-$ function [3]), a strong compensation between the Maki-Thompson (MT) and density-of-states (DOS) contributions to the conductivity occurs [2], while the Aslamazov-Larkin (AL) paraconductivity survives unchanged [4].

In a normal metal the transition from the diffusive to the quasi-ballistic regime is controlled by the relation between the electron elastic mean free path ℓ and the diffusive length $l_T = \sqrt{D/T}$ ($D = v_F^2\tau/2$ is the diffusion coefficient) or, in other words, by the ratio of the temperature T and the elastic scattering rate. This ratio is given by the value of $T\tau$, where τ is the elastic scattering time. In a superconductor, due to the presence of the additional length scale ξ_0, the range of impurity concentrations of a clean metal ($\xi_0 \ll \ell \Leftrightarrow T\tau \gg 1$) close to the superconducting transition temperature can be divided into clean ($\xi_0 \ll \ell \ll \xi_{GL}(T)$) and ultra-clean ($\xi_{GL}(T) \ll \ell$) regimes. In terms of the reduced temperature scale, the narrow range $\epsilon \ll 1/(T\tau)^2$ (which includes both the diffusive and clean regimes) can still be described by the local fluctuation theory, while the study of the fluctuation-dominated transport in the most interesting temperature interval $1/(T\tau)^2 \ll \epsilon \ll 1$ (ultra-clean regime) requires a nonlocal treatment. This may be particularly relevant in the case of the high-temperature cuprates, where $T\tau$ for the high-quality samples is estimated to reach 5–10, and almost all the range of the experimentally accessible reduced temperature ϵ belongs to the ultra-clean limit.

Indeed in a sufficiently clean superconductor ($T\tau \gg 1$), the locality condition $\epsilon \ll 1/(T\tau)^2$ almost contradicts to the $2D$ thermodynamical Ginzburg-Levanyuk criterion of the mean-field approximation applicability ($Gi \sim \frac{T_c}{E_F} \ll \epsilon$). Moreover, as it is known, the high order corrections for the transport coefficients become comparable with the mean-field results much before than those for the thermodynamical ones, namely at [5] $\epsilon \sim \sqrt{Gi}$. Hence, being interested in the study of the fluctuations in clean superconductors, *de facto*, one can speak only about their nonlocal behavior.

Electrical transport is affected by the presence of a perpendicular magnetic field. In particular, the fluctuation magnetoconductivity of a superconductor close to the transition temperature has been widely studied in the literature within the approximation of the local theory (see for example [6–8]). In this paper, we extend the calculation to the ultra-clean limit where the nonlocal fluctuation theory is required.

As it is well known, the relation between the value of the electron mean free path and the Larmour radius R_L is crucial for the study of the one-electron magnetotransport characteristics of a metal. For weak magnetic fields, ($\ell \ll R_L = v_F/\Omega_L$, where Ω_L is the Larmour frequency) the dominant effect consists simply in the bending of the quasi-classical trajectories, while for strong fields ($R_L \ll \ell$), the electron motion becomes considerably localized and the quasi-classical picture is no longer valid. The condition $\Omega_L\tau \sim 1$ separates the different regimes in the magnetoconductivity of a metal in the presence of impurities [9]. This condition can be also rewritten in terms of the reduced magnetic field $h = H/H_{c2}$

$$\Omega_L\tau = DeH\tau = \frac{v_F^2 e\tau^2 \Phi_0}{4\pi\xi_0^2}h = \frac{8\pi^2}{7\zeta(3)}(\tau T)^2 h \sim 1, \quad (1)$$

where $\Phi_0 = \pi/e$ is the magnetic flux and $H_{c2} = \Phi_0/2\pi\xi_0^2$ is the upper critical field. Then the regions of weak and strong fields for the one-electron magnetotransport can be written as $h \ll 1/(T\tau)^2$ and $1/(T\tau)^2 \ll h$.

When considering the effect of the magnetic field on the fluctuation-Cooper-pair transport, a discussion of the relevant length scales must include, side by side with R_L and ℓ, the GL coherence length $\xi_{GL}(T)$ as well. As a result, the weak fields regime, characterized by the ballistic motion of the Cooper pairs and the quasi-classical motion of the electrons, ($\xi_{GL}(T) \ll \ell \ll R_L$), can be defined as $h \lesssim 1/(T\tau)^2 \ll \epsilon \ll 1$, whereas, in the regime of strong fields $1/(T\tau)^2 \ll \min\{h, \epsilon\} \ll 1$, the one-electron motion can no longer be described by the bending of the quasi-classical trajectories only. Below we will present the results for the electrical conductivity as a function of both the reduced temperature and magnetic field strength for arbitrary values of the parameter $T\tau$.

The layout of the paper is as follows. In the next section we derive the general expressions for the fluctuation-induced electrical conductivity due to the quantum fluctuation processes (DOS and MT) valid for any impurity concentration. In Section 3 we perform the calculation of the electrical magnetoconductivity in the nonlocal limit. Section 4 contains a brief summary and a discussion of the results.

2 General expression for fluctuation conductivity

The electrical conductivity is given by the electromagnetic response kernel [10,11]:

$$\sigma = \lim_{\omega\to 0}\frac{Q^R(\omega)}{-\mathrm{i}\omega}\cdot \quad (2)$$

The Feynman diagrams which contribute to $Q^R(\omega)$ in the first order of perturbation theory in the fluctuations are shown in Figure 1.

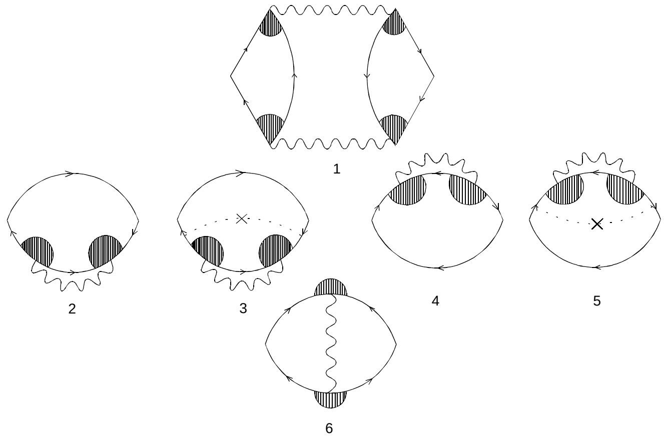

Fig. 1. Feynman diagrams for the leading-order contributions to $Q(\omega_\nu)$.

Here the wavy lines are the fluctuation propagators, L, the solid lines are the impurity-averaged normal-state Green's functions, G, the shaded semicircles are the vertex corrections arising from the impurities (cooperons), $\mathcal{C}$, the dashed lines with central crosses are additional impurity renormalizations. Without any approximation regarding the $T\tau$ parameter and in the absence of magnetic field, G, L and $\mathcal{C}$ for a two-dimensional (2D) superconductor are given by [2]:

$$G(\mathbf{p}, \varepsilon_n) = \frac{1}{\mathrm{i}\widetilde{\varepsilon}_n - \xi(\mathbf{p})} \quad (3)$$

$$L^{-1}(\mathbf{q}, \Omega_k) = -N_0\left\{\ln\frac{T}{T_c} + \sum_{n=0}^{\infty}\left[\frac{1}{n+\frac{1}{2}} - \frac{1}{\sqrt{\left(n+\frac{1}{2}+\frac{\Omega_k}{4\pi T}+\frac{1}{4\pi T\tau}\right)^2+\frac{v_F^2q^2}{16\pi^2T^2}}-\frac{1}{4\pi T\tau}}\right]\right\}$$

$$\mathcal{C}(\mathbf{q}, \varepsilon_1, \varepsilon_2) = \left\{1 - \frac{\Theta(-\varepsilon_1\varepsilon_2)}{\tau\sqrt{(\widetilde{\varepsilon}_1 - \widetilde{\varepsilon}_2)^2 + v_F^2q^2}}\right\}^{-1} \quad (4)$$

where, following the standard notation, $\widetilde{\varepsilon}_n = \varepsilon_n + \frac{\mathrm{sign}\varepsilon_n}{2\tau}$, $\xi(\mathbf{p}) = p^2/2m - \mu$, N_0 is the density of states and $\Theta(x)$ is the Heaviside function. The current vertices are

$$\mathbf{j}(\mathbf{p}, \varepsilon_n, \varepsilon_{n+\nu}) = e\mathbf{v}(\mathbf{p}) \quad (5)$$

where the $\mathbf{v}(\mathbf{p})$ is the velocity of quasiparticle. The quasi-classical Aslamazov-Larkin contribution (diagram 1 of Fig. 1) to the electrical conductivity of a 2D superconductor above T_c, was shown to be independent on the impurity concentration for all scattering regimes [2,12]. We restrict our analysis to the vicinity of the critical temperature, where as it is well known (see, for instance, Ref. [13])

$$Q^{\mathrm{DOS+MT}}(\omega_\nu) = Q_a(\omega_\nu) + Q_b(\omega_\nu) + Q_c(\omega_\nu) \tag{6}$$

$$Q_a(\omega_\nu) = \sum_q L(\mathbf{q},0)\, 2T^2 \sum_{\varepsilon_n} j^2(\mathbf{p},\varepsilon_{2n+\nu})\mathcal{C}^2(\mathbf{q},\varepsilon_{n+\nu},-\varepsilon_{n+\nu}) I_a(\mathbf{q},\varepsilon_n,\omega_\nu) \tag{7}$$

$$Q_b(\omega_\nu) = \sum_q L(\mathbf{q},0)\, 2T^2 \sum_{\varepsilon_n} j^2(\mathbf{p},\varepsilon_{2n+\nu})\mathcal{C}^2(\mathbf{q},\varepsilon_{n},-\varepsilon_{n}) I_b(\mathbf{q},\varepsilon_n,\omega_\nu) \tag{8}$$

$$Q_c(\omega_\nu) = \sum_q L(\mathbf{q},0)\, 2T^2 \sum_{\varepsilon_n} j(\mathbf{p},\varepsilon_{2n+\nu}) j(\mathbf{q}-\mathbf{p},\varepsilon_{2n+\nu})\mathcal{C}(\mathbf{q},\varepsilon_{n+\nu},-\varepsilon_{n+\nu})\mathcal{C}(\mathbf{q},\varepsilon_{n},-\varepsilon_{n}) I_c(\mathbf{q},\varepsilon_n,\omega_\nu) \tag{9}$$

$$I_a(\mathbf{q},\varepsilon_n,\omega_\nu) = \frac{2\pi N_0}{(\widetilde{\varepsilon}_{n+\nu}-\widetilde{\varepsilon}_n)^2}\left[\frac{1}{\sqrt{(2\widetilde{\varepsilon}_{n+\nu})^2+v^2q^2}} - \frac{\Theta(\varepsilon_n\varepsilon_{n+\nu})}{\sqrt{(\widetilde{\varepsilon}_{n+\nu}+\widetilde{\varepsilon}_n)^2+v^2q^2}} + \frac{2\widetilde{\varepsilon}_{n+\nu}\omega_\nu}{\left[(2\widetilde{\varepsilon}_{n+\nu})^2+v^2q^2\right]^{3/2}}\right] \tag{10}$$

$$I_b(\mathbf{q},\varepsilon_n,\omega_\nu) = \frac{2\pi N_0}{(\widetilde{\varepsilon}_{n+\nu}-\widetilde{\varepsilon}_n)^2}\left[\frac{1}{\sqrt{(2\widetilde{\varepsilon}_{n})^2+v^2q^2}} - \frac{\Theta(\varepsilon_n\varepsilon_{n+\nu})}{\sqrt{(\widetilde{\varepsilon}_{n+\nu}+\widetilde{\varepsilon}_n)^2+v^2q^2}} - \frac{2\widetilde{\varepsilon}_{n}\omega_\nu}{\left[(2\widetilde{\varepsilon}_{n})^2+v^2q^2\right]^{3/2}}\right] \tag{11}$$

$$I_c(\mathbf{q},\varepsilon_n,\omega_\nu) = -\frac{2\pi N_0 \mathrm{sign}(\varepsilon_n\varepsilon_{n+\nu})}{(\widetilde{\varepsilon}_{n+\nu}-\widetilde{\varepsilon}_n)^2}\left[\frac{1}{\sqrt{(2\widetilde{\varepsilon}_{n+\nu})^2+v^2q^2}} + \frac{1}{\sqrt{(2\widetilde{\varepsilon}_{n})^2+v^2q^2}} - \frac{2\Theta(\varepsilon_n\varepsilon_{n+\nu})}{\sqrt{(\widetilde{\varepsilon}_{n+\nu}+\widetilde{\varepsilon}_n)^2+v^2q^2}}\right] \tag{12}$$

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the static approximation for diagrams 2–6 is valid, and hence the Cooper pair bosonic frequency Ω_k can be set to zero. As a result the contribution of DOS and MT terms may be written in the form:

see equations (6–9) above

where Q_a comes from diagrams 4 and 5; Q_b comes from diagrams 2 and 3 (DOS-type terms); and Q_c comes from diagram 6 (MT-type term). Here

see equations (10–12) above

represent the integral over momentum $\mathbf{p}$. Let us stress that equations (6–12) describe the quantum fluctuation-induced electric conductivity in the case of any scattering regime.

To proceed further and to calculate analytically the conductivities one has to distinguish the different regimes of elastic electron scattering. In the diffusive and clean regimes one can use the local form (obtained by a low-q expansion) of both the fluctuation propagator and the cooperon

$$L(\mathbf{q},0) = -\frac{1}{N_0}\frac{1}{\xi^2q^2+\epsilon}; \quad \mathcal{C}(\mathbf{q},\varepsilon_1,\varepsilon_2) = \frac{|\tilde{\varepsilon}_1-\tilde{\varepsilon}_2|}{Dq^2+|\varepsilon_1-\varepsilon_2|}.$$

The positive coefficient ξ entering the fluctuation propagator is given by

$$\xi^2 = -\frac{v_F^2\tau^2}{2}\left[\psi\left(\frac{1}{2}+\frac{1}{4\pi\tau T}\right) - \psi\left(\frac{1}{2}\right) - \frac{1}{4\pi\tau T}\psi'\left(\frac{1}{2}\right)\right], \tag{13}$$

whereas in the ultra-clean or ballistic regime one has to use the full non-local form as given in equations (4).

Once we have performed the integration over p, we can proceed with the summation over the fermionic Matsubara frequency ε_n and analytical continuation $\omega_\nu \to -\mathrm{i}\omega$. For the electrical conductivity this procedure allows one to reproduce from the general equations (6–12) the limiting local [6] and non-local [2] cases. In the latter case the strong compensation between DOS and MT diagrams occurs and the total quantum correction to the conductivity tends to zero in the limit $T\tau \to \infty$ leaving the AL paraconductivity as the only effect [2].

The calculation of the fluctuation correction in the ballistic regime is not trivial and was first performed for the case of the electric conductivity in reference [2]. In this case one has to use the general form of L and C. To evaluate the contribution to the magnetoconductivity it is useful first to make an expansion in ω_ν up to the first order and then to expand each term of the expression in ω_ν in powers of $\frac{1}{\tau}$ up to the third order. One then verifies that the zero order in ω_ν vanishes. The resulting series has the following form

$$\mathcal{I}(q,\omega_\nu) = \omega_\nu \sum_{k=-2}^{\infty}\left(\frac{1}{T\tau}\right)^k {}^k(q). \tag{14}$$

One can see that although each diagram contributes to $\mathcal{I}^k(q)$ for $k \geq -2$ the sum gives a non vanishing contributions only for $k = 0, 1, 2...$

To generalize the result to the important case of a layered superconductor one has to make the substitution $\ln(1/\epsilon) \to 2\ln\left[2/\left(\sqrt{\epsilon}+\sqrt{\epsilon+r}\right)\right]$ where r is an anisotropy parameter[6] and to multiply 2D conductivity by $1/p_F$.

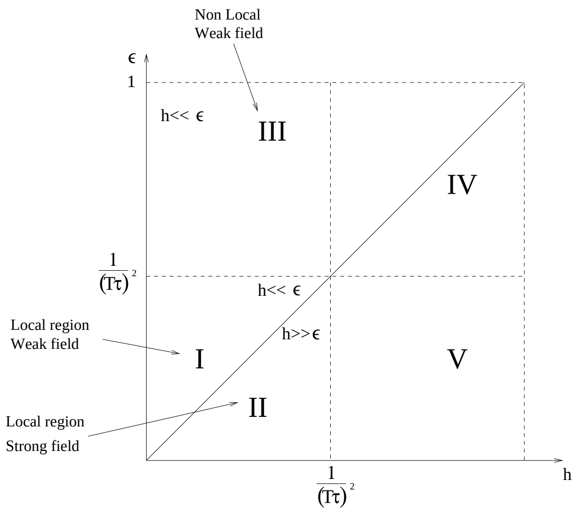

Fig. 2. The different regimes in fluctuation magnetotransport. See explanation in the text.

3 Fluctuation magneto conductivity in perpendicular magnetic field

To discuss the behavior of the transport coefficients in the presence of a magnetic field, it is convenient to represent all the different regimes in a $\epsilon - h$ plane as shown in the phase diagram of Figure 2. The bottom left quadrant, characterized by the condition $\epsilon, h \ll 1/(T\tau)^2$, can be divided into the domains I ($h \ll \epsilon$) and II ($h \gg \epsilon$), corresponding to the weak and strong field limits of the local fluctuation transport, respectively. The domain III ($\epsilon \gg 1/(T\tau)^2$, $h \ll \epsilon$) corresponds to the low field non-local situation, which was not studied before. It will be the main subject of the following discussion.

The fluctuation magnetoconductivity of a superconductor close to the transition temperature has been largely studied within the approximation of the local theory (see for example [6–8]). In order to present the complete picture, the asymptotic expressions for the different fluctuation contributions to the magnetoconductivity $\varDelta\sigma\left(\epsilon, h\right) = \sigma\left(\epsilon, h\right) - \sigma\left(\epsilon, 0\right)$ of a 2D superconductor in the local regime (domains I and II) are shown in the Table 1 side by side with the new results of the present paper valid in the non-local case (domain III).

Let us compare the values of the different contributions to the conductivity at the upper limit of validity of the local fluctuation theory (the line separating the domains I and III). One can easily see that, at $\epsilon \sim (T\tau)^{-2}$, the magnetic field dependent part of the DOS-MT$^{(reg)}$ contribution becomes of the order of the AL term but is opposite in sign.

The situation with the MT$^{(\mathrm{an})}$ contribution is complicated by its dependence on the pair-breaking. The phase-breaking time τ_ϕ is determined by the value of the magnetic field and other inelastic scattering processes (phonons, paramagnetic impurities, etc.):

$$\frac{1}{\tau_\phi} = \varOmega_L + \frac{1}{\tau_s},$$

so that for the phase breaking rate γ_ϕ one can write [6]:

$$\gamma_\phi = \frac{7\zeta(3)}{2^4\pi^2 T\tau_\phi}\frac{1}{T\tau} = \frac{\pi}{2}h + \frac{7\zeta(3)}{16\pi^2}\frac{1}{T^2\tau\tau_s}\cdot \qquad (15)$$

In the weak fields domain I, the first term in (15) is negligible with respect to ϵ, whereas the second one defines $\gamma_{\phi 0} \ \sim (T\tau)^{-2}\,\tau/\tau_s \ll (T\tau)^{-2}$ $(\tau_s \gg \tau)$. Hence for small ϵ the anomalous MT term can be omitted and the fluctuation magnetoconductivity is determined by the most singular ($\propto \epsilon^{-3}$) AL contribution only. *Vice versa*, at the upper limit of the validity of the local fluctuation theory (domain I), where $\epsilon \sim (T\tau)^{-2}$, one finds that the pair-breaking is certainly weak ($\gamma_{\phi 0} \ll \epsilon$). The corresponding anomalous MT contribution to the magnetoconductivity is of the same sign as the AL one but exceeds it by a factor of the order of $(\tau_s/\tau)^2$:

$$\varDelta\sigma^{\mathrm{MT(an)}} \sim \varDelta\sigma^{AL}\left(\frac{\tau_s}{\tau}\right)^2\cdot \qquad (16)$$

At the same time we know [14] that in the nonlocal limit $\epsilon \gtrsim (T\tau)^{-2}$ the MT contribution does not depend anymore on the pair-breaking, so that the extrapolation of (16) to the regime III has not to be taken too seriously. Moreover, a strong compensation of the DOS and MT zero-field contributions has been found for the case of the non-local fluctuations [2]. This is why below we will revise the problem of the fluctuation magnetoconductivity of a $2D$ superconductor in the non-local regime.

The magnetic field is supposed to be oriented perpendicular to the superconducting plane. In our calculation we will follow the common scheme used in the local case [5]: the effect of a magnetic field on the fluctuation conductivity is formally taken into account by the quantization of the Cooper pair center-of-mass motion. This means that instead of the integration over the long wavelength fluctuation contributions one has to sum the contributions of different Landau levels which classify the fluctuation Cooper pairs states in a magnetic field.

In the non-local fluctuation regime, we can still use the same idea as long as the one-particle motion can be considered as quasi-classical and the effect of the electron trajectories bending between successive impurity scattering events can be neglected in comparison with the Cooper pairs spectrum quantization. This situation takes place in the domain III of the phase diagram (Fig. 2).

The peculiarity of the non-local fluctuation dynamics, when the modes with momenta $q \geq \ell^{-1}$ are involved, requires the dealing with the cooperon and fluctuation propagator given in the general form of equations (4). They are valid for any $q \ll p_F$(one can see that for $v_F q \ll \max\{T, \tau^{-1}\}$ they reduce to the well known local expressions [5]). The non-local electromagnetic response functions for DOS and MT diagrams in the absence of magnetic field were obtained in reference [2], where the strong cancellation between them occurred. Namely, the regular part of the MT diagram appearing from summation in diagram 6 of the Matsubara interval of frequency $n \in \left]-\infty, -\nu - 1\right]$ and $n \in [0, \infty]$ exactly compensate the

Table 1. The fluctuation contributions to the electrical magnetoconductivity for domains I, II and III.

	I $h \ll \epsilon$	II $\epsilon \ll h$	III $h \ll \epsilon$
$\Delta\sigma^{AL}$	$-\frac{e^2}{2^5}\frac{h^2}{\epsilon^3}$	$-\sigma^{AL}(0,\epsilon)+\frac{e^2}{8}\frac{1}{h}$	$-\frac{e^2}{2^5}\frac{h^2}{\epsilon^3}$
$\Delta(\sigma^{\rm DOS}+\sigma^{\rm MT(reg)})$	$\frac{e^2\pi(T_c\tau)^2}{3\,2^5 7\zeta(3)}\frac{h^2}{\epsilon^2}$	$\frac{e^2 2\pi(T_c\tau)^2}{7\zeta(3)}\ln\frac{h}{4\epsilon}$	$0.13\frac{e^2}{T_c\tau}\frac{h^2}{\epsilon^2}$
$\Delta\sigma^{\rm MT(an)}$ $\epsilon \ll \gamma_\varphi$	$-\frac{e^2}{3\times 2^5}\frac{h^2}{\gamma_\varphi\epsilon^2}$	$-\sigma^{\rm MT}(0,\epsilon)+\frac{e^2}{16}\frac{1}{\gamma_\varphi}\ln\frac{\gamma_\varphi}{8h}$	
$\Delta\sigma^{\rm MT(an)}$ $\gamma_\varphi \ll \epsilon$	$-\frac{e^2}{3\times 2^5}\frac{h^2}{\epsilon\gamma_\varphi{}^2}$	$-\sigma^{\rm MT}(0,\epsilon)+\frac{3\pi^2 e^2}{32s}\frac{1}{h}$	$-0.002\frac{e^2}{T_c\tau}\frac{h^2}{\epsilon^2}$

result of corresponding summation in the diagrams 2–5 (DOS Type) leaving only:

$$Q^{\rm DOS}(\omega_\nu)+Q^{\rm MT(reg)}(\omega_\nu)=\frac{2^5\pi T}{\tau^2}\sigma_D\int\frac{{\rm d}\mathbf{q}}{(2\pi)^2}L(\mathbf{q},0)T\sum_{n=-\nu}^{-1}\frac{1}{\left[(2\varepsilon_n)^2+(v_Fq)^2\right]^2}. \quad (17)$$

The resting anomalous MT contribution

$$Q^{\rm MT(an)}(\omega_\nu)=-\frac{2^4\pi T}{\tau^2}\sigma_D\int\frac{{\rm d}\mathbf{q}}{(2\pi)^2}L(\mathbf{q},0)T\sum_{n=-\nu}^{-1}\frac{(4\varepsilon_n)^2+(v_Fq)^2}{\left[(2\varepsilon_n)^2+(v_Fq)^2\right]^3}, \quad (18)$$

where σ_D is the conductivity of a 2D normal metal.

The quantization of the Cooper pair motion in a magnetic field for the clean case can be carried out in the same way as in dirty one. By introducing the gauge invariant momentum operator $\widehat{\mathbf{q}}\to\nabla/{\rm i}-2e\mathbf{A}$, we replace, q^2 in the fluctuation contributions evaluated in the absence of magnetic field by $2h\,(n+1/2)\,/\xi_0^2$. As a result one has to replace summation over Landau levels instead of the momentum integration:

$$\int\frac{{\rm d}^2\mathbf{q}}{(2\pi)^2}\to\frac{H}{\Phi_0}\sum_m=\frac{h}{2\pi\xi_0^2}\sum_m.$$

In the domain of weak magnetic fields (III) one can carry out this summation by means of the Euler-Maclaurin's transformation

$$\sum_{k=0}^{K}f(k)=\int_{-1/2}^{K+1/2}f(k){\rm d}k-\frac{1}{24}\left[f'(K+1/2)-f'(-1/2)\right]. \quad (19)$$

Being interested in the magnetoconductivity $\Delta\sigma\,(\epsilon,h)=\sigma\,(\epsilon,h)-\sigma\,(\epsilon,0)$ one can see that the integral, corresponding to the zero field conductivity, is cancelled out and the problem is reduced to the calculation of the derivatives of the zero-field expressions (17–18). Taking into account the fast convergence to zero in ε_n it is possible to use the decomposition

$$\sum_{n=-\nu}^{-1}f(\varepsilon_n)=\sum_{n=0}^{\infty}\left(f(\varepsilon_{n-\nu})-f(\varepsilon_n)\right).$$

In this way we avoid the presence of ω_ν in the sum limits making the corresponding function analytical in this variable [5]. Then the analytical continuation of (17–18) becomes trivial: it is sufficient to substitute $\omega_\nu\Rightarrow-{\rm i}\omega\to0$ and to expand $f(\varepsilon_n-{\rm i}\omega)$ in ω before the summation:

$$\sum_{n=-\nu}^{-1}f(\varepsilon_n)|_{\omega_\nu\Rightarrow-{\rm i}\omega\to0}=-{\rm i}\omega\sum_{n=0}^{\infty}\frac{16\varepsilon_n}{\left((2\varepsilon_n)^2+(v_Fq)^2\right)^3}.$$

The quantization of the Cooper pair motion results in:

$$\sigma^{\rm DOS}(\epsilon,h)+\sigma^{\rm MT(reg)}(\epsilon,h)=-\frac{h}{2^3\pi^5N_0\xi_0^2}\frac{\sigma_D}{\tau^2T^3}\times\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{(n+1/2)}{\left((n+1/2)^2+\frac{2}{7\zeta(3)}h\,(2k+1)\right)^3}\times\frac{1}{\epsilon+\sum\limits_{n=0}^{\infty}\left(\frac{1}{n+1/2}-\frac{1}{\sqrt{\left(n+\frac{1}{2}\right)^2+\frac{2}{7\zeta(3)}h(2k+1)}}\right)}. \quad (20)$$

The sum over k is performed applying the Euler-Maclaurin's transformation (19). Appearing integral determines the fluctuation conductivity in the zero field and it is canceled out from $\Delta\sigma(\epsilon,h)$. The last summation in equation (20) can be performed in terms of Euler ζ-function, what leads to the following expression for the magnetoconductivity in the region III:

$$\Delta\sigma^{\rm DOS}(\epsilon,h)+\Delta\sigma^{\rm MT(reg)}(\epsilon,h)=\frac{31}{21\pi^2}\frac{\zeta(5)}{\zeta(3)}\frac{e^2}{T_c\tau}\frac{h^2}{\epsilon^2}=0.13\frac{e^2}{T_c\tau}\frac{h^2}{\epsilon^2}.$$

The analogous treatment of the anomalous MT contribution yields the same functional dependence but with a coefficient two orders of magnitude smaller than the DOS-MT$^{(reg)}$ one:

$$\Delta\sigma^{\rm MT(an)}(\epsilon,h)=-0.002\frac{e^2}{T_c\tau}\frac{h^2}{\epsilon^2}.$$

Nevertheless, one can see that both these contributions turn out to be negligible in comparison with the more singular AL magnetoconductivity

$$\Delta\sigma^{AL}(\epsilon,h)\approx -0.03e^2\frac{h^2}{\epsilon^3}.$$

4 Discussion

The analysis proposed demonstrates that for weak fields the fluctuation-induced non-local electric magnetoconductivity is determined by the AL contribution practically over all the range of the impurity concentrations. The MT anomalous contribution can be observed just in the case of a very weak pair-breaking in the local regime and far enough from the critical temperature while the strong compensation of the DOS with the MT terms in the non-local regime makes their contribution negligible.

Let us discuss qualitatively the hierarchy of the different fluctuation contributions in the high field region when h exceeds $1/(T\tau)^2$ (domains IV and V of Fig. 2). In this range of magnetic fields the quantization of the quasiparticle motion becomes important and the quasi-classical approximation for the one-particle Green function is no longer valid [9]. In the case of weak fields taking into account the bending of the quasiparticle trajectories may only yield corrections to the conductivity which are quadratic in the magnetic field, but not singular in $T-T_c$. For this reason they were omitted with respect to the Cooper pairs contribution. Upon increasing the reduced magnetic field, h, above $1/(T\tau)^2$ (domains IV and V), the quasiparticles become localized by the magnetic field as well. The integration over electron momenta produces additional powers of h in the denominator in direct analogy with the case of the magnetoconductivity of a normal metal [9]. The resulting DOS and MT corrections to the magnetoconductivity will therefore rapidly fall with the magnetic field as $h^{-\alpha}$ where $\alpha \geq 2$. The AL paraconductivity of a clean superconductor in a magnetic field has been calculated in [12] for an arbitrary $\Omega_L\tau$. In the strong-field limit $\Omega_L\tau \gg 1$ the AL paraconductivity behaves as h^{-2}.

Let us stress the qualitative difference between the magnetic field dependencies of the fluctuation magnetoconductivity in the ballistic and diffusive limits: in the former case the unique AL survived contribution vanishes for fields larger than $H_{c2}(\epsilon)$ as the power H^2, while in the latter the remaining DOS contribution is very robust with respect to magnetic-field effects and decreases logarithmically [7] disappearing only at the second critical field $H_{c2}(0)$.

In conclusion, we have studied the superconducting fluctuation contributions to the electric conductivity of a two-dimensional and layered superconductor in the quasi-ballistic regime, including the effect of an applied perpendicular magnetic field. We have derived the general expressions for the quantum corrections to the fluctuation transport, valid for both local and non-local cases, and shown their consistency with the previously obtained results [2,6].

Regarding the fluctuation-induced electrical magnetoconductivity, we have demonstrated that the non-local contributions from the MT and DOS processes are negligible in comparison with the AL term, which is insensitive to the impurity scattering. The hierarchy of the fluctuation contributions in the high field limit has been also discussed and demonstrated to be qualitatively different from the local case.

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3. We use, for the sake of simplicity, a unit system with $c = \hbar = k_B = 1$
4. We recall that the AL contribution has a quasi-classical interpretation, and may be seen as the direct contribution to the current of the fluctuating Cooper pairs. The MT and DOS terms, in contrast, are purely quantum effects and therefore are much more sensitive to the degree of disorder or impurity concentration. One way of looking at the physics behind the DOS and MT terms is to take into account that in the presence of superconducting fluctuations the system is composed of "normal" and "superconducting" regions. The "superconducting" regions affect the conductivity of the "normal" regions via the well-known proximity effect or, to use the quasiparticle language, the Andreev scattering. The DOS and MT terms describe, therefore, the suppression of the density of states and the leakage of the pair wave function in the "normal" regions
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